Table 2 Values of λ_{cr} and Ω_{cr} for a simply supported and clamped panel

L/t	γ	λ_{cr}	Ω_{cr}
50.0	0.0	474.6	1726
	1.0	474.6	1727
	10.0	474.6	1736
	100.0	474.6	1826
	1000.0	474.6	2726
25.0	0.0	465.8	1694
	1.0	465.8	1695
	10.0	465.8	1704
	100.0	465.6	1793
	1000.0	465.6	2689
10.0	0.0	410.2	1491
	1.0	410.2	1492
	10.0	410.2	1501
	100.0	409.5	1588
	1000.0	403.3	2460
5.0	0.0	279.6	1006
	1.0	279.6	1007
	10.0	279.2	1014
	100.0	276.8	1094
	1000.0	252.4	1888

Table 3 Values of λ_{cr} and Ω_{cr} for a clamped panel

L/t	γ	λ_{cr}	Ω_{cr}
50.0	0.0	625.6	2687
	1.0	625.6	2688
	10.0	625.6	2697
	100.0	625.6	2787
	1000.0	625.6	3687
25.0	0.0	606.9	2610
	1.0	606.9	2611
	10.0	606.9	2620
	100.0	606.6	2709
	1000.0	605.8	3604
10.0	0.0	499.1	2154
	1.0	499.1	2155
	10.0	499.1	2164
	100.0	498.4	2250
	1000.0	492.0	3120
5.0	0.0	290.7	1246
	1.0	290.7	1247
	10.0	290.5	1253
	100.0	288.0	1332
	1000.0	263.8	2122

The matrix $[F]$ is obtained by assembling the element matrices $[f]$ derived from the energy expression $\bar{U}$ given by

$$\bar{U}=\frac{\gamma}{2}\int_0^l w^2\mathrm{d}x \tag{5}$$

where l is the element length.

It should be noted that the approximate aerodynamic theory used here is valid only for $M_\infty > 1.5$.

Numerical Results

Equation (1) is solved by using any standard algorithm to obtain the eigenvalues and eigenvectors. In the present study for a given γ and L/t (where t is the thickness of the panel), an eight-element idealization is used that gives accurate results for the flutter problems as shown in Ref. 7. The value of γ_{cr} is obtained where the two lowest nondimensional eigenvalue parameters, Ω_1 and Ω_2, coalesce to Ω_{cr}. This is repeated to obtain λ_{cr} and Ω_{cr} for different values of γ and L/t.

Using the procedure described above, values of λ_{cr} and Ω_{cr} at γ of 0, 1.0, 10.0, 100.0, and 1000.0 and L/t of 50.0, 25.0, 10.0, and 5.0 are obtained for simply supported, simply supported and clamped, and clamped panels. The results are presented in Tables 1-3.

Conclusions

Based on the present study of the supersonic flutter of panels resting on an elastic foundation, the following conclusions can be drawn:

1) For a given L/t and γ, the values of λ_{cr} and Ω_{cr} increase as one goes from simply supported to simply supported and clamped and to clamped panels.

2) For thin panels (i.e., for $L/t=50$), λ_{cr} is the same for any value of γ and Ω_{cr} increases by the value of γ compared to the value of Ω_{cr} for $\gamma=0$.

3) For a given γ, as L/t decreases, the values of λ_{cr} and Ω_{cr} decrease.

4) For thicker panels, for a given L/t the values of λ_{cr} decrease as γ increases. Thus, for thicker panels, the additional foundation stiffness has a destabilizing effect.

5) Even for thicker panels with L/t as low as 5, the value of λ_{cr} remains constant up to $\gamma=1.0$ and the shift in Ω_{cr} is equal to the value of γ compared to the value Ω_{cr} for $\gamma=0.0$.

6) The effect of shear deformation and rotatory inertia is significant for values of $L/t<10.0$.

7) All the above conclusions hold for the three types of boundary conditions considered in the present study.

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Thermal Resistance of Circular Cylinder Cross Sections with Convective and Flux Prescribed Boundaries

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Introduction

In the thermal analysis of fuel rod interaction with containment tubing,[1] in the analysis of double-walled heat exchangers which employ longitudinally grooved tubing as

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one of the members,[2] and in the analysis of the heat loss from the hulls of ships,[3] the geometrical and thermal configuration arises wherein thermal energy enters the inner or outer surface of a member over only a portion of that surface, flows through the member, and leaves through the oposite radial surface where a convective condition for the surface heat exchange is operative. This Note examines the conduction heat transfer problem associated with the configuration just described with the purpose of determining the thermal resistance of such a member under conditions of steady, two-dimensional heat transfer.

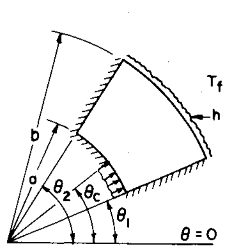
The analysis is restricted to the case of evenly spaced contact locations such that a convenient single typical cell can be extracted for analysis purposes. This typical cell is delineated by the planes of thermal symmetry existing along the radial line passing through the center of the flux distribution and along the radial line passing through the mid-point between neighboring contact locations. The heat flow over the contact region is modeled as a flux distribution over the region of contact of the member with its mating surface. The problem geometry is then that shown in Fig. 1a for a typical cell. A uniform heat transfer coefficient h is applied to the opposite surface and provides the thermal interaction of this surface with an environment at a temperature of T_f.

The special case of this problem for which the inner and outer radii become large, reverting the problem to that of planar, two-dimensional, steady heat transfer through a rectangular cross section, has been examined by several investigators.[3-8]

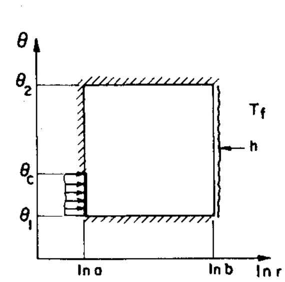
Most recently, Schneider et al.[7] systematically examined the rectangular cross section problem. In their paper the thermal constriction resistance was determined and a comprehensive range of parameters were examined with the results presented in graphical form. This latter paper has provided a comprehensive examination of the rectangular problem. However, the situation in which the shape of the conducting member is cylindrical has seen little attention. [9,10] The solution will be formulated for the case of an arbitrary flux distribution over the contact, and two distributions in addition to the uniform flux case will be specifically considered.[7] The three distributions examined will be useful in estimating limits for the thermal resistance by which most cases of practical interest will be bounded. The thermal resistance is presented for all three cases as a function of the geometric parameters and the heat transfer coefficient h through the Biot modulus.

Problem Solution

The domain is bounded between the inner and outer radii, a and b respectively and between two values of the angular coordinate, θ_1 and θ_2. The contact is defined by the additional angle θ_c, where the heat flow surface extends from θ_1 to θ_c. The solution can accommodate readily the interchanging of the thermal specifications on the inner and outer surfaces. The ambient temperature is denoted by T_f and thermally interacts with the convective surface through the uniform coefficient h.



a) Original domain.



b) Transformed domain.

Fig. 1 Problem geometry.

Considering steady-state heat transfer with no internal heat generation, the governing differential equation, Laplace's equation, is written in circular cylinder coordinates as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0 \tag{1}$$

and prior to effecting the solution to Eq. (1), it is useful to utilize the ideas presented in a previous paper by Schneider and Yovanovich.[8] In that paper, it was noted that if a transformed coordinate system was defined according to the rules pertaining to conformal mapping procedures, the governing differential equation is Laplace's equation in a Cartesian-like form. Thus, the transformation is used that

$$u_1 \equiv \ln r \tag{2}$$

The governing equation, Eq. (1), then becomes

$$\frac{\partial^2 T}{\partial u_1^2}+\frac{\partial^2 T}{\partial \theta^2}=0 \tag{3}$$

and the geometry of the transformed domain, in the $\ln r$-θ plane, is simply rectangular in cross section as illustrated in Fig. 1b.

To simplify the problem in the transformed domain, the transformed coordinates are normalized by the reference difference, $\theta_2-\theta_1$. Thus, the definitions are introduced that

$$\xi \equiv \frac{\ln r-\ln a}{\theta_2-\theta_1}=\frac{\ln r/a}{\theta_2-\theta_1} \tag{4}$$

$$\eta \equiv \frac{\theta-\theta_1}{\theta_2-\theta_1} \tag{5}$$

Further, the temperature distribution is nondimensionalized according to the definition

$$T^* \equiv [kL(T-T_f)]/Q \tag{6}$$

where

$$Q=aL\int_{\theta_1}^{\theta_c} q(\theta)\,\mathrm{d}\theta \tag{7}$$

is the total heat flow rate through the contact region, L is the length of the cross section (i.e., into the page length), and k, the thermal conductivity, assumed constant throughout the domain. The differential equation then becomes

$$\frac{\partial^2 T^*}{\partial \xi^2}+\frac{\partial^2 T^*}{\partial \eta^2}=0 \tag{8}$$

and the transformed boundary conditions become

$$\eta=0;\quad 0\le\xi\le\alpha;\quad \frac{\partial T^*}{\partial \eta}=0 \tag{9a}$$

$$\eta=1;\quad 0\le\xi\le\alpha;\quad \frac{\partial T^*}{\partial \eta}=0 \tag{9b}$$

$$\xi=\alpha;\quad 0\le\eta\le 1;\quad \frac{\partial T^*}{\partial \xi}=-BiT^*(\alpha,\eta) \tag{9c}$$

$$\xi=0;\quad 0\le\eta\le\epsilon;\quad \frac{\partial T^*}{\partial \xi}=-q^*$$

$$\epsilon<\eta\le 1;\quad \frac{\partial T^*}{\partial \xi}=0 \tag{9d}$$

where the geometric and thermal parameters have been introduced. These are

$$\alpha \equiv \frac{\ell n(b/a)}{\theta_2 - \theta_1} \tag{10}$$

$$\epsilon \equiv \frac{\theta_c - \theta_1}{\theta_2 - \theta_1} \tag{11}$$

$$Bi \equiv hb(\theta_2 - \theta_1)/k \tag{12}$$

$$q^* \equiv qaL(\theta_2 - \theta_1)/Q \tag{13}$$

The preceding problem is mathematically equivalent to that solved by Schneider et al.[1] and reference can be made directly to that solution. The final form of the nondimensional temperature distribution is then

$$T^* = \alpha - \xi + 1/Bi + \sum_{n=1}^{\infty} C_n \cos(n\pi\eta) \times [\sinh(n\pi\xi) - \psi_n \cosh(n\pi\xi)] \tag{14}$$

where

$$\psi_n = \left[\frac{n\pi\cosh(n\pi\alpha) + Bi\sinh(n\pi\alpha)}{n\pi\sinh(n\pi\alpha) + Bi\cosh(n\pi\alpha)}\right] \tag{15}$$

and

$$C_n = -\frac{2}{n\pi}\int_0^{\epsilon} q^*(\eta)\cos(n\pi\eta)\,d\eta; \quad n = 1,2,3,\ldots \tag{16}$$

Following the usual definition, the overall thermal resistance is given in dimensionless form by

$$R^* \equiv RkL = kL(\bar{T}_c - T_f)/Q \tag{17}$$

where $\bar{T}_c$ is the average temperature of the contact region. Evaluating Eq. (17) yields the result

$$R^* = \alpha + \frac{1}{B_i} - \sum_{n=1}^{\infty} \frac{C_n \psi_n}{n\pi\epsilon} \sin(n\pi\epsilon) \tag{18}$$

A dimensionless thermal constriction resistance can be obtained by subtracting from the overall thermal resistance, the resistance of the circular section which results from one-dimensional heat conduction from the surface $\xi = 0$ to the fluid at temperature T_f. The one-dimensional resistance is given in nondimensional form by

$$R^*_{1-D} = \alpha + 1/Bi \tag{19}$$

so that the nondimensional constriction resistance is given by

$$R^*_c = -\sum_{n=1}^{\infty} \frac{C_n \psi_n}{n\pi\epsilon} \sin(n\pi\epsilon) \tag{20}$$

where three flux distributions are considered. These are given by

$$q(\eta) = q_0(1-u^2)^{\mu}; \quad \mu = -½, 0, +½ \tag{21}$$

where u is defined by

$$u \equiv (\theta - \theta_1)/(\theta_c - \theta_1) \tag{22}$$

The corresponding values of C_n for use in Eq. (20) are given in Table 1 for the three cases.

Table 1 Influence of μ on C_n

μ	C_n
$-½$	$(-2/n\pi) J_0(n\pi\epsilon)$
0	$(-2/n^2\pi^2\epsilon)\sin(n\pi\epsilon)$
$+½$	$(-4/n^2\pi^2\epsilon) J_1(n\pi\epsilon)$

Through the transformation of coordinates, the mathematical problem and, hence, the result defined by Eq. (20) for the constriction resistance is identical to that examined by Schneider[7] for the rectangular problem. Thus, the reader is referred to that reference for a graphical representation of the constriction resistance.

Discussion and Conclusions

The thermal constriction resistance of a circular cylinder section has been obtained. One of the constant radius surfaces is convectively coupled to an ambient temperature while over a portion of the opposite surface a flux distribution is applied. Three specific flux distributions have been explicitly considered.

By posing the problem, for this two-dimensional, steady, zero source strength case, in a conformally mapped transformation domain, it was possible to treat the problem as an equivalent Cartesian-like problem. It was found that the influence of the cylindrical geometry appears directly in the Biot modulus, which, for the case in which the outer surface is convectively coupled to T_f, contains the value of outer surface arc length, $b(\theta_2 - \theta_1)$. Of course, due to the transformation employed, the geometric parameter contains the ratio of the outer to inner radius, through the logarithm of this ratio.

In considering the case in which the outer radius has the imposed flux distribution over a portion of its surface while the inner surface is convectively coupled to an ambient temperature T_f, the modifications required to the given solution are minimal. It can be demonstrated easily that the only change required is that the Biot modulus be redefined using the arc length at the inner surface as the characteristic dimension instead of that at the outer surface. Thus, $a(\theta_2 - \theta_1)$ replaces $b(\theta_2 - \theta_1)$ in the defining equation for Bi. With this single adaptation, all of the results presented herein also apply for this reverse situation.

Acknowledgments

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An Efficient Numerical Method for Solving Inverse Conduction Problem in a Hollow Cylinder

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Nomenclature

A,B,C,D	= coefficient of square temperature matrix
Bi	= Biot number, hL/k
h	= heat-transfer coefficient
k	= thermal conductivity
L	= thickness of material $(r_0 - r_i)$
$\dot{q}_c$	= surface heat flux
r	= radial coordinate
r_i	= inner radius of cylinder
r_o	= outer radius of cylinder
T	= nondimensional temperature
T^{n-1}	= nondimensional temperature at beginning of time step
T^n	= nondimensional temperature at end of time step
t	= nondimensional time, $\alpha\tau/L^2$
Δt	= computing time
X	= nondimensional radial coordinate, $(r-r_i)/L$
ΔX	= node thickness
θ	= temperature
α	= thermal diffusivity
τ	= time

Subscripts

g	= combustion gas temperature
i,o,R	= node identifier
in	= initial temperature
j	= thermocouple location
s	= surface temperature

Superscript

n	= designated point $(t+\Delta t)$

Introduction

THE calculation of surface heat flux and surface temperature from a measured temperature history at some location inside the body is called the inverse heat conduction problem. Many configurations such as spheres, slabs, and cylinders have been studied and methods such as numerical, graphical, series, convolution integral, and Laplace transforms have been utilized. An excellent discussion of previous investigations for solving the inverse problem can be found in Ref. 1. In a previous study,[2] the two-level Crank-Nicholson method was used for estimation of heat-transfer coefficient in a rocket nozzle of a finite slab thickness. But, as mentioned in Ref. 3, the equivalent slab treatment is not valid for the thickness-to-radius ratio exceeding 0.2, which is case in the vicinity of the throat region of a rocket nozzle, and always gives conservative estimates for temperatures in the cylindrical structure. This Note extends the inverse conduction problem to the cylindrical geometry using a grid point shift arrangement[4] on the boundaries. The grid point shift of surface conditions contains the following advantages as compared to implicit analog of the boundary conditions:

1) The arrangement of a grid point shifted from the boundaries is the most convenient to solve the inverse problem in cylindrical or spherical coordinates.

2) Using this arrangement, one can easily simulate mathematically and physically the most general boundary condition of the third kind.[5] It should be mentioned here that the number of unknown values of temperatures for a given number of increments is independent of the boundary conditions.

3) The surface temperature oscillation does not occur when first interior point is located one-half increment from the boundary, whereas a grid point located on the boundary requires a backward finite difference analog and also needs care in selecting the time interval in order to achieve a stable solution.

One disadvantage of the grid point shift arrangement is that the temperature at the boundaries is evaluated as the average of the exterior value and the last interior value. The exterior value can be obtained from the appropriate boundary condition analog.

This Note reports a simple numerical scheme to solve the inverse conduction problem using transient temperature data for estimating the unknown surface conditions. A general digital program is discussed that can treat a variety of boundary conditions using a single set of equations.

Analysis

Consider a long hollow cylinder with a finite wall thickness, having a heat sink at one surface and a perfect insulation at the other. The material of the cylinder is considered to be homogeneous and isotropic with constant thermophysical properties. Let r_i and r_o be, respectively, the inner and outer radii. If the temperature of the cylinder is initially uniform at θ_{in}, the mathematical problem governing the temperature may be written as

$$\frac{\partial^2 T}{\partial X^2} + \frac{1}{X+(r_i/L)}\frac{\partial T}{\partial X} = \frac{\partial T}{\partial t} \tag{1}$$

with the boundary conditions

$$\frac{\partial T(0,t)}{\partial X} = -1, \qquad t>0 \tag{2}$$

$$\frac{\partial T(1,t)}{\partial X} = 0, \qquad t>0 \tag{3}$$

and

$$T(X,0) = 0 \qquad \text{for all } X \tag{4}$$

where nondimensional temperature T is $k(\theta-\theta_{in})/\dot{q}_c L$.

The finite difference conditions of Eq. (1) with the boundary conditions of Eqs. (2) and (3) are developed by arranging the grid points as described in Ref. 4. The value of

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